

QUANTUM ALGEBRA IN THE MIXED LIGHT PSEUDOSCALAR MESON STATES

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Abstract

In this paper, we investigate the entanglement degrees of pseudoscalar meson states via quantum algebra $Y(su(3))$. By making use of transition effect of generators J of $Y(su(3))$, we construct various transition operators in terms of J of, and act them on $\eta\text{-}\pi^0\text{-}\eta'$ mixing meson state. The entanglement degrees of both the initial state and final state are calculated with the help of entropy theory. The diagrams of entanglement degrees are presented. Our result shows that a state with desired entanglement degree can be achieved by acting proper chosen transition operator on an initial state. This sheds new light on the connect among quantum information, particle physics and Yangian algebra.

Keywords: Yangian $Y(su(3))$; $\eta\text{-}\pi^0\text{-}\eta'$ mixing; the entanglement degree

PACS numbers: 02.20.-a, 03.65.-w, 21.65.-f

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I. INTRODUCTION

It has been realized that quantum entanglement is a key ingredient in quantum computation, quantum communication, and quantum cryptography[1]. Many two-level quantum systems, or qubits, have been widely used in quantum information[2, 3]. Recently, the bipartite qutrit systems have drawn people's attention and exhibited varieties of advantages. It enables powerful computation[4], establishes secure communication[5] and cryptography[6], and reduces the communication complexity[7]. Additionally, the high energy quantum teleportation using neutral kaons has been investigated[8]. These hint the possibility of connecting quantum information and particle physics, this connection may reveal novel and interesting feature. Considering these two aspects, we take the mixed light pseudoscalar meson states, a type of bipartite qutrit system, into account.

In particle physics, the study on mixed meson states[9] plays an important role and many works are devoted to this field[10–12], especially the study on $\eta - \eta'$ mixed state, since it provides a unique opportunity for testing QCD [13] which is widely used in describing strong interaction. In the last decades, people have studied the mixing angle[14], the hadronic η - η' decay[15], η - η' mixing in radiative ϕ -meson decay[16], and so on. However, much work of particle physics has been done for testing the theory and experiment while hardly concerned with quantum information and the related algebra configuration, such as Yangians. As is known, the light pseudoscalar meson states in quark-flavor basis have $su(3)$ symmetry[17]. It is important to expand the research on the symmetry of pseudoscalar meson states to Yangian $Y(su(3))$.

Yangian, as an algebra beyond the Lie algebra, is a powerful mathematical method for investigating the new symmetry of quantum systems which are nonlinear and integrable. Demonstrating and investigating whether simple physical systems possess Yangian is important and helpful for exploring physical systems via quantum algebra. People have found the Yangian symmetry in many physical models, such as Calogero-Sutherland model[18], the Hubbard model[19] and the Heisenberg model[20], etc. The realizations of $Y(sl(2))$ [21, 22] as the simplest one in Yangian algebra have been gained much attention. However, the Yangian related to the Lie algebra $su(3)$, $Y(su(3))$, which is closed to the light pseudoscalar meson states in particle physics, need to be investigated in more detail. Thus, much attention is going to be payed in this paper to the application of $Y(su(3))$ algebra in the meson

systems here.

II. $Y(su(3))$ ALGEBRA IN η - π^0 - η' MIXING SYSTEM

In the low mass hadron region, the violations of isospin symmetry for pseudoscalar meson states within QCD are generated by the admixtures of η - π^0 - η' . η and η' are linear combination of $su(3)$ singlet $\eta^{0'}$ and octet η^0 , π^0 is another $su(3)$ octet. Because of the important application and significance of η - π^0 - η' mixing, we choose their superposition states as the initial state

$$|\phi\rangle = \alpha_1|\eta^{0'}\rangle + \alpha_2|\pi^0\rangle + \alpha_3|\eta^0\rangle, \quad (1)$$

where α_1 , α_2 and α_3 are the normalized real amplitudes and they satisfy $\alpha_1^2 + \alpha_2^2 + \alpha_3^2 = 1$. $|\eta^{0'}\rangle = \frac{\sqrt{3}}{3}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$, $|\eta^0\rangle = \frac{\sqrt{6}}{6}(-|u\bar{u}\rangle - |d\bar{d}\rangle + 2|s\bar{s}\rangle)$, $|\pi^0\rangle = \frac{\sqrt{2}}{2}(|u\bar{u}\rangle - |d\bar{d}\rangle)$.

The entanglement degree of the genuine N-particle qutrit pure state[23] is measured by its mean entropy

$$C_{\Phi}^{(N)} = \begin{cases} \frac{1}{N} \sum_{i=1}^n S_{(i)} & \text{if } S_i \neq 0 \forall i \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where $S_i = -Tr((\rho_{\Phi})_i Log_3(\rho_{\Phi})_i)$ is the reduced partial Von Neumann entropy for the i th particle only, with the other $N - 1$ particles traced out, and $(\rho_{\Phi})_i$ is the corresponding reduced density matrix. Since the system in Eq. (1) is bipartite qutrit, i.e. $N = 2$, the corresponding entanglement degree of this initial state can be calculated via Eq.(2), thus we have

$$\begin{aligned} C_{\phi} = & -\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)^2 Log_3\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)^2 \\ & -\left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)^2 Log_3\left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)^2 \\ & -\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3\right)^2 Log_3\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3\right)^2. \end{aligned} \quad (3)$$

The behavior of C_{ϕ} depending on α_1 and α_2 (due to the normalization condition, α_3 is not an independent parameter.) is given in Fig. 1.

Before constructing transition operators, let us first give a brief introduction of Yangian $Y(su(3))$. $Y(su(3))$ algebra[24] is generated by the generators $\{I^a, J^a\}$ which are usually

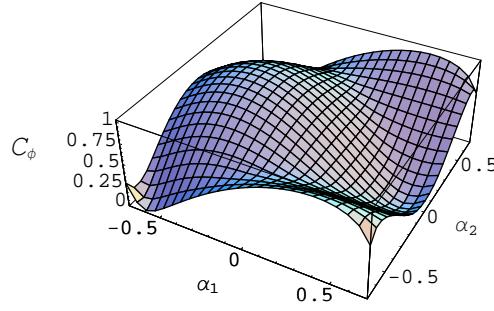


FIG. 1: (color online). The entanglement degree of the initial state $|\phi\rangle$ for different α_1 and α_2 .

defined as follows

$$\begin{aligned} I^a &= \sum_i F_i^a, \\ J^a &= \mu I_1^a + \nu I_2^a + \frac{i}{2} \lambda f_{abc} \sum_{i \neq j} \omega_{ij} I_i^b I_j^c \quad (i, j = 1, 2). \end{aligned} \quad (4)$$

Here I^a form a $su(3)$ algebra characterized by f_{abc} , $\{F_i^a, a = 1, \dots, 8\}$ form a local $su(3)$ on the i site, and are equal to half of the corresponding Gell-Mann matrices, μ, ν, λ are parameters or Casimir operators and $\omega_{ij} = -\omega_{ji}$ which satisfies

$$\omega_{ij} = \begin{cases} 1 & i > j \\ -1 & i < j \\ 0 & i = j \end{cases}. \quad (5)$$

A more practical expression is expressed as

$$\bar{I}^\pm = J^1 \pm i J^2, \quad \bar{U}^\pm = J^6 \pm i J^7, \quad \bar{V}^\pm = J^4 \pm i J^5, \quad \bar{I}^3 = J^3, \quad \bar{I}^8 = \frac{2}{\sqrt{3}} J^8. \quad (6)$$

Here J^a ($a = 1, \dots, 8$) are the generators of $Y(su(3))$.

Due to the transition effect of Yangian generators, transition operators P can be constructed as compositions of the generators in Eq. (6). P can be looked upon as a function of $\bar{I}^\pm, \bar{U}^\pm, \bar{V}^\pm, \bar{I}^3$ and \bar{I}^8 , namely, $P = F[\bar{I}^\pm, \bar{U}^\pm, \bar{V}^\pm, \bar{I}^3, \bar{I}^8]$. When acting the transition operator on $|\phi\rangle$ in Eq. (1), a final state $|\phi'\rangle = P|\phi\rangle$ can be gotten and its entanglement degree $C_{\phi'}$ can also be calculated out via Eq.(2). Different final states with desired entanglement degrees thus can be gotten by acting corresponding transition operators on initial states.

In order to illustrate this issue clearly, several simple examples are going to be discussed in more detail.

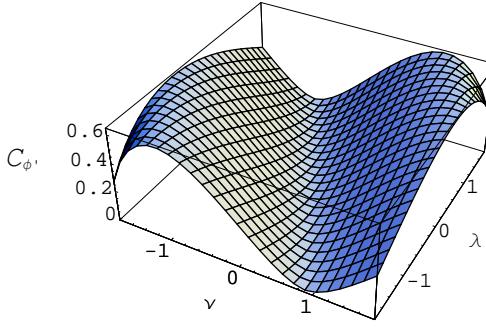


FIG. 2: (color online). The entanglement degree of the final state $|\phi'\rangle$ varies with ν and λ when $\alpha_1 = \alpha_2 = \frac{1}{2}$.

III. OBTAINING DIFFERENT ENTANGLEMENT DEGREES BY TUNING TRANSITION OPERATORS

As we discussed before, the entanglement degree of final state can be tuned by changing the parameters of the transition operator. As an example, let us act the transition operator of $P = \bar{V}^+ + \bar{V}^-$ on the initial state in Eq.(1), the corresponding final state is

$$|\phi'\rangle = P|\phi\rangle = (\nu + \frac{\lambda}{2})[(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)|K^+\rangle + (\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)|K^-\rangle]. \quad (7)$$

The normalizing condition leads to $\mu = \frac{\lambda}{2}$ and $(\nu + \frac{\lambda}{2})^2[1 - (\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2] = 1$. As discussed in the preceding section, the entanglement degree of the final state $|\phi'\rangle$ can be calculated by the use of Eq.(2) as

$$C_{\phi'} = -(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 \text{Log}_3(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 - (\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)^2 \text{Log}_3(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)^2. \quad (8)$$

As we can see, by changing the value of ν and λ , the entanglement degree $C_{\phi'}$ can be tuned. This behavior is shown in Fig. 2 for the case of $\alpha_1 = \alpha_2 = \frac{1}{2}$.

Another more simpler case is $P = \bar{I}^8$ for which the final state reads

$$|\phi'\rangle = P|\phi\rangle = \frac{1}{3}(\nu + \frac{\lambda}{2})[-\frac{\sqrt{2} + 2\sqrt{6}}{3}\alpha_3|\eta^0'\rangle + \alpha_2|\pi^0\rangle - (\sqrt{2}\alpha_1 + \alpha_3)|\eta^0\rangle] \quad (9)$$

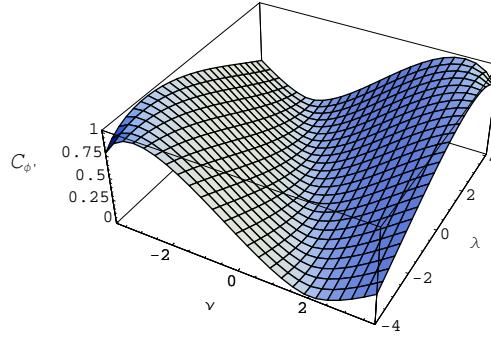


FIG. 3: (color online). The entanglement degree of the final state $|\phi'\rangle$ varies with ν and λ when $\alpha_1 = \alpha_2 = \frac{1}{2}$.

with normalization condition $\mu = \frac{\lambda}{2}$ and $(\nu + \frac{\lambda}{2})^2[1 + 3(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)^2] = 3$. Similarly,

$$C_{\phi'} = -\frac{1}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 \text{Log}_3 \frac{1}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 - \frac{1}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 \text{Log}_3 \frac{1}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3)^2 - \frac{4}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)^2 \text{Log}_3 \frac{4}{9}(\nu + \frac{\lambda}{2})^2(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3)^2, \quad (10)$$

which also depends on ν and λ , as shown in Fig. 3.

These two examples clearly shows that by tuning the parameters ν and λ of transition operators, the entanglement degree of final state varies between 0 and 1.

However, there are exceptional cases in which the entanglement degree of final state is found to be the same as the initial state or be zero, independent with ν and λ , as is shown in the next section.

IV. TWO EXCEPTIONAL CASES

Although generally, transition operators with different ν and λ provide final states with different entanglement degrees, there do exist special cases in which the final state possesses the same entanglement degree with the initial state or the final state are totally disentangled. Both cases are very important in quantum information and quantum computation.

For the transition operator $P = \bar{I}^- + \bar{U}^- + \bar{V}^-$, the final state is

$$|\phi'\rangle = P|\phi\rangle = -\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)|\pi^+\rangle + \left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)|K^0\rangle + \left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3\right)|K^-\rangle \quad (11)$$

with normalizing condition $\mu + \nu = 1$ and $\mu = \frac{\lambda}{2}$. It is very easy to verify that the entanglement degree of the final state $|\phi'\rangle$ is equal to the one of the initial state, namely, $C_{\phi'} = C_\phi$.

Same thing happens on transition operator $P = \bar{I}^+ + \bar{U}^+ + \bar{V}^+$, corresponding to whom the final state reads

$$|\phi'\rangle = P|\phi\rangle = -\left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)|K^+\rangle + \left(\frac{\sqrt{3}}{3}\alpha_1 - \frac{\sqrt{2}}{2}\alpha_2 - \frac{\sqrt{6}}{6}\alpha_3\right)|\pi^-\rangle + \left(\frac{\sqrt{3}}{3}\alpha_1 + \frac{\sqrt{6}}{3}\alpha_3\right)|\bar{K}^0\rangle \quad (12)$$

with normalizing condition being $\mu + \nu = 1$ and $\mu = \frac{\lambda}{2}$. Again, calculation shows that $C_{\phi'} = C_\phi$.

Without any difficulty, it can be verified that transition operators $P = \bar{I}^- + \bar{V}^+$, $P = \bar{I}^+ + \bar{U}^-$, and $P = \bar{U}^+ + \bar{V}^-$ lead to total disentangled final states, independent of the choice of α_1 and α_2 in the initial state.

V. CONCLUSIONS

In conclusion, we have investigated the entanglement degrees of pseudoscalar meson states via quantum algebra $Y(su(3))$. By making use of transition effect of generators J of $Y(su(3))$, we have constructed various transition operators in terms of J , and have acted them on $\eta\pi^0$ - η' mixing meson state. The entanglement degrees of both the initial state and final state have been calculated with the help of entropy theory. Our result shows that a state with desired entanglement degree can be achieved by acting proper chosen transition operator on an initial state. This is very helpful to control the degree of entanglement in quantum communication. Entanglement has been considered as an essential resource in most applications of quantum information[1]. Although the quantum channels used in quantum teleportation are usually represented by a maximally entangled pair[25], partially entangled quantum channel becomes a hot topic[26] nowadays due to the noise factors in the realistic

world. Moreover, in quantum computing, a new important development is the probabilistic implementation of a nonlocal gate by using a single non-maximally entangled state[27]. The success fidelity maintains a high level though the state shared is partially entangled.

We believe that our work provide a connection among particle physics, quantum information and quantum algebra, and the result may shed new light on entanglement controlling in quantum computing.

[1] A. Galindo and M. A. M. Delgado, *Rev. Mod. Phys.* **74** (2002) 347.

[2] F. G. Deng, G. L. Long and X. S. Liu, *Phys. Rev. A* **68** (2003) 042317.

[3] H. Lu and G. C. Guo, *Phys. Lett. A* **276** (2000) 209.

[4] A. M. Childs and I. L. Chuang, *Phys. Rev. A* **63** (2000) 012306.

[5] N. K. Langford, R. B. Dalton, M. D. Harvey, J. L. Ó'Brien, G. J. Pryde, A. Gilchrist, S. D. Bartlett and A. G. White, *Phys. Rev. Lett.* **93** (2004) 053601.

[6] H. B. Pasquinucci and A. Peres, *Phys. Rev. Lett.* **85** (2000) 3313.

[7] Č. Brukner, M. Żukowski and A. Zeilinger, *Phys. Rev. Lett.* **89** (2002) 197901.

[8] Y. Shi, *Phys. Lett. B* **641** (2006) 75.

[9] M. Gell-Mann and A. Pais, *Phys. Rev.* **97** (1955) 1387.

[10] T. Feldmann and P. Kroll, *Phys. Rev. D* **58** (1998) 114006

[11] A. Magiera and H. Machner, *Nucl. Phys. A* **674** (2000) 515

[12] P. Kroll, *Modern Phys. Lett. A* **20** (2005) 2667.

[13] V. Dmitrasinovic, *Phys. Rev. D* **56** (1997) 247.

[14] J. Cao, F. G. Cao, T. Huang and B. Q. Ma, *Phys. Rev. D* **58** (1998) 113006.

[15] B. Borasoy and R. Nilbler, *Phys. J. A* **26** (2005) 383.

[16] F. D. Fazio and M. R. Pennington, *J. High Energy Phys.* **7** (2000) 51.

[17] M. Y. Han and Y. Nambu, *Phys. Rev.* **139** (1965) B1006.

[18] D. Uglov, *Commun. Math. Phys.* **191** (1998) 663.

[19] A. Kundu, *Phys. Lett. A* **249** (1998) 126.

[20] D. Bernard, *Inter. J. Modern Phys. B* **7** (1993) 3517.

[21] F. D. M. Haldane, *Phys. Rev. Lett.* **60** (1988) 635.

[22] B. S. Shastry, *Phys. Rev. Lett.* **60** (1988) 639.

- [23] F. Pan, D. Liu, G. Y. Lu and J. P. Draayer, *Phys. Lett. A* **336** (2005) 384.
- [24] M. L. Ge. and K. Xue, *Yang-Baxter Equation* (Shanghai: Shanghai Scientific and Technical Publishers) (1999) p511.
- [25] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W. K. Wootters, *Phys. Rev. Lett.* **70** (1993) 1895.
- [26] J. X. Fang, Y. S. Lin, S. Q. Zhu and X. F. Chen, *Phys. Rev. A* **67** (2004) 014305.
- [27] L. Chen and Y. X. Chen, *Phys. Rev. A* **71** (2005) 054302.